

Due: Beginning of Class Monday April 6, 2009.

1. Let S be the statement,

“If $x + y$ is prime, then x is prime or y is prime.”

a. The contrapositive of S is, _____

b. The converse of S is, _____

2. Write out a proof of the following statement about the integers using only the definitions of even and odd together with algebra.

If x is even and y is odd, then $x + y$ is odd.

3. Evaluate each expression.

a. $\text{power}(\{\emptyset, a, b\}) =$ _____

b. $\{a, b, c, d\} - (\{a, b, c\} \cap \{b, c, d, e\}) =$ _____

c. $\{a, b, c\} - (\{a, b, c, d\} - \{c, d, e\}) =$ _____

d. $\{a, b, c\} \oplus \{a, b, c\} =$ _____

e. (Let \mathbf{N} be the universe). $\{1, 2, 3\}' - \{3, 4, 5\}' =$ _____

f. $\{a, b\} \times \{1, 2, 3\} =$ _____

g. $\text{head}(\langle b, c, d \rangle) =$ _____

h. $\text{tail}(\langle a, b, c, d, e, f \rangle) =$ _____

i. $\text{cons}(b, \langle c, b, a \rangle) =$ _____

4. For each integer n let $A_n = \{x \mid x \in \mathbf{N} \text{ and } x \leq n\}$. Evaluate each of the following expressions.

a. $A_8 \cap A_3 =$ _____

b. $A_8 \cup A_3 =$ _____

c. $A_8 - A_3 =$ _____

d. $A_3 - A_8 =$ _____

5. Given the following facts about three sets A , B , and C .

$$|A \cap (B \cup C)| = 100,$$

$$|A \cap B| = 70,$$

$$|A \cap C| = 80,$$

Find $|A \cap B \cap C|$.

6. Solve the language equation for L .

$$\{a, b, ba\}L = \{a, ba, b, ab, bb, bab\}.$$

7. Let L and M be two languages and let $x \in L^*(ML)$. Describe the general form of x by writing it as a concatenation of strings, where each string is in either L or M .

8. Calculate the number of strings over the alphabet $\{a, b, c, d\}$ that have length 7, begin with a , b , or c , and contain at least one d .

9. Draw a minimal spanning tree for the following weighted graph.

