

Due: Beginning of Class Monday April 13, 2009.

1. Evaluate each expression.
 - a. $\text{floor}(x) \neq \text{ceiling}(x)$ if and only if _____
 - b. $\text{gcd}(236, 112) =$ _____
 - c. $(-12) \bmod 7 =$ _____
 - d. $\log_2(1024) =$ _____
 - e. $\text{map}(\text{gcd}, \text{dist}(2, \text{seq}(4))) =$ _____
 - f. If $f(x) = x \bmod 5$, then $\text{map}(f, \text{seq}(6)) =$ _____

2. Let $f : \mathbf{N}_9 \rightarrow \mathbf{N}_9$ be defined by $f(x) = 3x \bmod 9$. Evaluate each expression.
 - a. $f^{-1}(\{0, 3\}) =$ _____
 - b. $f(\{1, 2, 4\}) =$ _____
 - c. $\text{range}(f) =$ _____

3. Let $f(x) = 3x^2$ and let $g(x, y) = 2x + y$. Find an expression that uses f and g to represent the following expression.

$$6a^2 + 3b^4.$$

4. Express each of the following function definitions as a **composition** of known functions from the set $\{\text{seq}, \text{dist}, \text{pairs}, \text{map}, +, -, *, \text{cons}, \text{head}, \text{tail}\}$.
 - a. $f(n, g) = \langle g(0), g(1), \dots, g(n) \rangle$.

 - b. $f(n) = \langle \log_2(1), \log_2(2), \dots, \log_2(n+1) \rangle$.

5. Let $f : \mathbf{N} \rightarrow \mathbf{N}$ be defined by $f(x) = x \bmod 12$.
 - a. Show that f is not surjective.

 - b. Show that f is not injective.

6. Show that the set S of odd integers and the set \mathbf{Z} of integers have the same cardinality. (i.e., find a bijection between the two sets.)

7. Let $f : \mathbf{N}_7 \rightarrow \mathbf{N}_7$ be defined by $f(x) = (4x + 3) \bmod 7$.

a. (Fill in the blank.) f is a bijection because $\gcd(\text{______}) = 1$.

b. Find a formula for f^{-1} , the inverse of f .

8. Let $S = \{\text{one, two, three, four, five, six, seven, eight}\}$ and suppose that $h: S \rightarrow \mathbf{N}_8$ is the hash function defined by

$$h(x) = \text{length}(x) \bmod 8,$$

where $\text{length}(x)$ is the number of letters in x . Use h to place each element of S into the following hash table starting with one, then two, and so on until eight. Resolve collisions by **linear probing with a gap of 3**.

0	
1	
2	
3	
4	
5	
6	
7	