

Due: Beginning of Class Monday May 4, 2008.

1. Find the partition induced by the following equivalence relation over the set \mathbf{N} .

$$a \sim b \quad \text{iff} \quad a \bmod 4 = b \bmod 4.$$

2. Let $x \sim y$ iff x and y are nonempty lists over $\{a, b\}$ with the same tail.

a. (Fill in the blank). The relation \sim is an equivalence relation because it is the kernel relation of _____

b. List the elements in each of the following equivalence classes.

$$[\langle a \rangle] = \underline{\hspace{10em}}$$

$$[\langle a, b \rangle] = \underline{\hspace{10em}}$$

$$[\langle a, a, b \rangle] = \underline{\hspace{10em}}$$

3. Let $f : \mathbf{N} \rightarrow \mathbf{N}$ be defined by $f(n) = \text{ceiling}(n/4)$. Describe the partition on \mathbf{N} induced by the kernel relation on f .

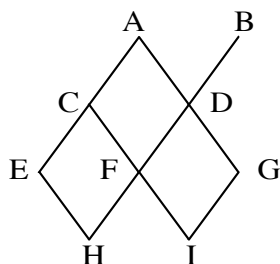
4. A graph with vertex set $\{a, b, c, d\}$ has its edges sorted by weight as follows:

$$\{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{a, d\}.$$

Use Kruskal's algorithm to find a minimal spanning tree T by showing the value of T and the corresponding equivalence classes at each step of the algorithm.

5. Let $D = \{2, 3, 6, 12, 24, 36\}$ and for any $x, y \in D$ let $x \prec y$ mean $x \mid y$ (i.e, x divides y). Draw the poset diagram for the partial order \prec on D .

6. Given the following poset diagram for the set $\{A, B, C, D, E, F, G, H, I\}$.



Find each of the following items, where $S = \{C, D, F\}$.

- The minimal elements of S .
 - The maximal elements of S .
 - The lower bounds of S .
 - The upper bounds of S .
 - The least upper bound of S .
 - The greatest lower bound of S .
7. Given the poset $\langle \mathbf{N} \times \mathbf{N}, \prec \rangle$, where $(a, b) \prec (c, d)$ means $a + b < c + d$. Write down a descending chain of maximum length that starts with $(3, 2)$.
8. Write an inductive proof that the following statement is true for all natural numbers n .

$$2 + 6 + 10 + \dots + (4n - 2) = 2n^2.$$

9. Write out an induction proof of the following equation for all $n \in \mathbf{N}$.

$$3 + 5 + 7 + \dots + (2n + 3) = (n + 1)(n + 3).$$