CS 340 Written Homework Assignment #5 **Name**___________________________
Due: Beginning of Class Monday May 11, 2009.

1. For any nonempty set S let $f : S \times \text{lists}(S) \to \mathbf{N}$ be defined as follows:

$f(x, L) =$ if $L = \langle\, \rangle$ then 0
else if $x = \text{head}(L)$ then $1 + f(x, \text{tail}(L))$
else $f(x, \text{tail}(L))$.

Let $P(L)$ = "$f(x, L)$ is the number of occurrences of x in L." Write out an induction proof that $P(L)$ is true for all lists L.

2. Find the unknown quantity for each of the following problems.

a. A solution to a problem has 63 possible outcomes. An algorithm to solve the problem has a ternary decision tree of depth d. What is the smallest value that d could be?

b. A solution to a problem has x possible outcomes. An algorithm to solve the problem has a binary decision tree of depth 5. What can the value of x be?

c. A solution to a problem has 100 possible outcomes. An algorithm to solve the problem has an n-way decision tree of depth 4. What is the smallest value that n could be?

3. Use known closed forms and summation facts to find a closed form for each of the following expressions:

a. $\sum_{i=0}^{n} (3i + 2)$

b. $2\sum_{i=1}^{n} 3^{i+1}$

4. Find an expression for the number of times, in terms of the natural number n, that S is executed in the following algorithm.

```
i := 0;
while i < n do
    j := i;
    while j < n do S; j := j + 1 od;
    i := i + 1
od
```

5. For each of the following problems, find an expression to represent the answer (DO NOT EVALUATE IT).

a. How many ways can 7 people be arranged in a row?

b. How many ways can 5 people be arranged in a row when the people are chosen from a set of 12 people?

c. How many different sets of 5 cans of soda that can be gotten from a machine that dispenses 4 kinds of soda?

d. How many strings of length 12 over $\{a, b, c, d\}$ contain 2 a's, 2 b's, 5 c's, and 3 d's?

6. Given the following procedure P defined for all natural numbers n.

```
P(n):   if n = 0 then
            C(0)
        else
            C(n);
            P(n – 1)
        fi
```

Let a_n denote the number of times that a certain operation is executed during the execution of $P(n)$. Suppose that $C(n)$ executes the operation $2n$ times. Write down a recurrence to describe a_n. Do not solve it.

7. Given the following recurrence.

$$a_0 = 1$$
$$a_n = 3a_{n-1} + 4n.$$

Solve the recurrence for a_n by cancellation or substitution. Leave the answer in summation form. You do not have to find a closed form for the answer.

8. Given the following recurrence.

$$a_0 = 1$$
$$a_1 = 2$$
$$a_n = 3a_{n-1} + 4a_{n-2}$$

Let $A(x) = \sum_{n=0}^{\infty} a_n x^n$. Use the generating function technique to find an equation for $A(x)$ that does not involve the summation. Stop when you get to where partial fractions are needed.

9. Suppose we have a recurrence with generating function $A(x) = \sum_{n=0}^{\infty} a_n x^n$ and we find that $A(x)$ has the form

$$A(x) = \frac{3}{x+2} + \frac{2}{1-3x}.$$

Find the closed form for a_n.