CS340 - Discrete Structures for Engineers

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Important Dates:

Exam 1 - Monday, April 20

Exam 2 - Wednesday, May 13

Final Exam - TBD, but likely during June 3 class time

Grading will be as follows:

Written Assignments (10%)

In-class Quizzes (10%) (yes, attendance is important!)

Two 2-hour exams (50%)

Final Exam (30%)

Study Habits

- Study the glossary of symbols and definitions to get to know the "language".
- Proficiency will come from doing problems. Go beyond the assigned work. Make up your own problems and solve them.
- Look at problems early so your subconscious has plenty of time to play with them.
- Review daily (all the way from the beginning).
- Read ahead before each lecture.
- Don't expect immediate success. Anything worthwhile takes time and effort.

Summary: Problem solving proficiency will be key to your success!

What is a Proof[1]?

Jury Trial - truth is ascertained by twelve people selected at random

Word of Boss - truth is ascertained from someone with whom it is unwise to disagree

Experimental Science - truth is guessed and confirmed or refuted by experimentation

Mathematics also has a specific notion of "proof" or way of ascertaining truth:

A *proof* is a demonstration that some statement is true.

We normally demonstrate proofs by writing English sentences mixed with symbols.

We'll consider statements that are either true or false. If *A* and *B* are statements,

not *A* - ***negation*** - opposite in truth value from A

A and *B* - ***conjunction*** - true exactly when both A and B are true

A or *B* - ***disjunction*** - true except when both A and B are false

if A then B - ***conditional statement*** - with *hypothesis A* and *conclusion B*. Its contrapositive is "if not B then not A" and its *converse* is "if B then A".

Statements with the same truth table are *equivalent*.

The table shows that a conditional and its contrapositive are equivalent.

A	B	if A then B	if not B then not A
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

1. Professor Albert Meyer, MIT

Proof Techniques

We'll give sample proofs about numbers. Here are some definitions.

integers: …, -2, -1, 0, 1, 2, …

odd integers: …, -3, -1, 1, 3, … (have the form $2k + 1$ for some integer k).

even integers:…, -4, -2, 0, 2, 4, … (have the form $2k$ for some integer k).

$m \mid n$ (read *m divides n*) if $m \neq 0$ and $n = km$ for some integer k.

p is prime if $p > 1$ and its only divisors are 1 and p.

Here are some of the characteristics of a good proof:

- It is clear and correct!
- It has a nice structure, like a good program. It is broken up into separate parts that define and prove key intermediate properties. This makes it easy to understand the reason the whole thing works. It also makes it more likely that pieces can be reused.
- Like a scientific experiment, someone else must be able to "replicate" (i.e. understand) your proof.

Exhaustive Checking

Some statements can be proven by exhaustively checking a finite number of cases.

Example 1: There is a prime number between 200 and 220.

Proof: Check exhaustively and find that 211 is prime. QED.

Example 2: Each of the numbers 288, 198, and 387 is divisible by 9.

Proof: Check that 9 divides each of the numbers. QED.

Conditional Proof

Most statements we prove are conditionals. We start by assuming the hypothesis is true. Then we try to find a statement that follows from the hypothesis and/or known facts. We continue in this manner until we reach the conclusion.

Example 3: If x is odd and y is even then $x - y$ is odd.

Proof: Assume x is odd and y is even. Then $x = 2k + 1$ and $y = 2m$ for some integers k and m. So we have

$$x - y = 2k + 1 - 2m = 2(k - m) + 1,$$

which is an odd integer since $k - m$ is an integer. QED.

Example 4. If x is odd then x^2 is odd.

Proof: Assume x is odd. Then $x = 2k + 1$ for some integer k. So we have

$$x^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1,$$

which is an odd integer since $2k^2 + 2k$ is an integer. QED.

Example 5: If x is even, then x^2 is even.

Proof: **Class Informal Quiz**

Example 6: If x^2 is odd, then x is odd

Proof: The contrapositive of this statement is "if x is even, then x2 is even," which is true by Example 5. QED.

Example 7: If x^2 is even, then x is even

Proof: **Class Informal Quiz**

If and Only If (Iff) Proofs

A statement of the form "A if and only if B" means "A implies B" and "B implies A." So there are actually two proofs to give. Sometimes the proofs can be written as a single proof of the form "A iff C iff D iff … iff B," where each iff statement is clear from previous information.

Example 8: x is even if and only if (iff) $x^2 - 2x + 1$ is odd.

Proof:	x is even iff $x = 2k$ for some integer k	(*definition*)
	iff $x - 1 = 2k - 1$ for some integer k	(*algebra*)
	iff $x - 1 = 2(k - 1) + 1$ for some integer $k - 1$	(*algebra*)
	iff $x - 1$ is odd	(*definition*)
	iff $(x - 1)^2$ is odd	(*examples 4 & 6*)
	iff $x^2 - 2x + 1$ is odd	(*algebra*) QED.

Proof By Contradiction

A false statement is called a contradiction. For example, "S and not S" is a contradiction for any statement S. A truth table will show us that "if A then B," is equivalent to "A and not B implies false." So to prove "if A then B," it suffices to assume A and also to assume not B, and then argue toward a false statement. This technique is called proof by contradiction.

Example 9. If x^2 is odd then x is odd.

Proof: Assume that x^2 is odd and x is even. Then x = 2k for some integer k.

So we have

$$x^2 = (2k)^2 = 4k^2 = 2(2k^2),$$

which is even since $2k^2$ is an integer. So we have x^2 is odd and x^2 is even, a contradiction. So the statement is true. QED.

Example 10. If 2 | 5n then n is even.

Proof: Assume that 2 | 5n and n is odd. Since 2 | 5n, we have 5n = 2d for some integer d. Since n is odd, we have n = 2k + 1 for some integer k. Then we have

$$2d = 5n = 5(2k + 1) = 10k + 5$$

So 2d = 10k + 5. Solve for 5 to get

$$5 = 2d - 10k = 2(d - 5k)$$

But this says that 5 is an even number, a contradiction. So the statement is true. QED.